Estimation of the Influence of Test Stimulus Precision on Test Quality for Parametric Faults in Analog Integrated Circuits

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With the upcoming trend towards built-in test structures and implicit testing, more and more issues of test design need to be resolved during circuit design. A basic requirement for manual as well as for automatic test generation is to assess how accurately a given test strategy will classify good and faulty circuits. Measurement error plays an important role here and must be taken into account to assess the precision of a test strategy. A second, often underestimated cause of error is the limited degree of accuracy of analog test stimulus generation.

In this paper, we propose a new method to evaluate and identify influences of test quality degradation due to measurement error and test stimulus variation. We will show that for analog testing, the precision of test stimulus generation and the precision of measuring the circuit’s response are of similar importance.

Keywords: analog test, measurement error, jitter.

1 Introduction

The increasing trend towards integration of digital and analog components on the same chip has spawned growing attention to the test needs of mixed-signal ICs. After all, the price of an increasing number of devices is presently dominated by the cost of production testing [1]. A major part of these testing costs are due to performance test of analog components.

Faults that occur in analog circuits are commonly classified into catastrophic faults (hard faults) caused e.g. by spot defects and parametric faults (soft faults) [2]. A circuit fails due to a parametric fault, if random fluctuations inherent to the manufacturing process lead to a significant performance loss and violation of the circuit’s specification. Mixed-signal test engineers and designers are primarily concerned with parametric faults because these faults are hard to distinguish from acceptable process variations. Catastrophic failures are most often detected by tests that were primarily designed for detecting parametric failures [3–6].

Most of the existing work dealing with test design for analog circuits can be classified into two groups. The first group uses measurements of the specified performances only, and aims at minimizing test cost by optimally ordering tests or by reducing the number of tests [3, 7]. The second group constructs an implicit test based on measurements that are regarded to be sensitive and reasonable, but do not necessarily include the specified performances of the circuit. Most of these approaches aim at the detection of catastrophic faults [8–10]. Recently, approaches aiming at the detection of parametric faults for linear time-invariant (LTI) systems [6, 11, 12] and for arbitrary circuits [4, 13, 14] have been published.

To evaluate the quality of an implicit test strategy for parametric faults, the influences of unidentifiable process variation (ambiguity groups), measurement error and imprecise input stimulus generation must be considered. Previous work on test selection for parametric faults focused either on ambiguity groups or on measurement error for tests that were not specification-driven but aimed at parameter estimation [15, 16]. For specification-driven test, statistical algorithms were used to integrate process variation and measurement noise into one analysis [4, 13]. In [6], the uncertainty of the prediction of single process parameters was used in a cost function for test selection. Monte Carlo analysis was then used on the set of selected tests to evaluate the influence of measurement error and stimulus variation found in commercial testers. This showed to nearly double the probability of misclassification for one example circuit in comparison to the ideal exact measurement and stimulus generation.

Research in the area of analog and mixed-signal built-in self test (BIST) showed results regarding on-chip support for generation of test stimuli as well as for evaluation of the DUT’s output signals [12, 17–19]. To circumvent the complexity of high-speed high-performance test equipment, some BIST strategies try to detect circuit faults by means of implicit testing performed by test circuitry that is integrated with the DUT [20, 21].

The application of BIST increases the need for test quality estimation with regard to process variation, measurement error, and stimulus variation for two reasons: Firstly, BIST strategies may aim at using simple test stimuli that can be generated very precisely, as well as performing measurements that are selected with regard to low measurement error. A better assessment of these influences may directly improve BIST solutions that are based on their minimization. Secondly, BIST circuitry is influenced by the same process variations that lead to the parametric faults that it should detect.

In Section 2, a model for testing is introduced that includes measurement error, process variation and test stimulus variation. Based on a first-order approximation of measurements and specified circuit performances, estimators of the specified performances are established. The accuracy of these estimators is then analyzed...
with regard to its sensitivity to measurement error and test stimulus variation. The generalized model can be applied to functional test, indirect test and to parameter estimation or fault analysis. In Section 3, the proposed methodology is applied to an example circuit and is compared to a Monte-Carlo simulation at transistor level.

2 Test model

Figure 1 shows the test model used in the following: A device under test (DUT) receives an input stimulus. The response of the device is measured and a vector of measurement results \( \mathbf{m} \), e.g. a certain number of sampled voltage values at the output node, is evaluated from the circuit response. The measurements \( \mathbf{m} \) will vary between circuits due to statistical fluctuations of device and process parameters like \( t_{\text{ox}} \) or \( V_{\text{th}} \) variation. These variations are modeled by a statistical distribution of device and process parameters \( s \sim N(0,\mathbf{C}_s) \) and they represent the source of device faults. The measurement result \( \mathbf{m} \) is then used to estimate a specified performance \( f \) of the circuit, e.g. slew rate or settling time. Based on the estimation \( \hat{f}(\mathbf{m}) \), the test decides whether the DUT should be accepted or rejected. In the following we will identify different sources of error that influence the quality of this estimation.

There are random variations in the test equipment that cause the test stimulus to vary between measurements. For example, the rise time of a square pulse test input signal or the frequency of a sine test input signal are subject to small statistical variations that are determined by the accuracy of the input signal generation. If a set of input signals is to be injected simultaneously at a set of circuit nodes, a random delay between the input signals may occur. These variations are modeled by an additional random vector of test input parameters \( \mathbf{\varepsilon}_t \sim N(0,\mathbf{C}_t) \).

Another cause of variation of \( \mathbf{m} \) is the finite precision of the measuring process that is determined by limitations in speed or resolution of the applied measurement equipment. This variation is modeled by an additional random vector \( \mathbf{\varepsilon}_m \sim N(0,\mathbf{C}_m) \).

A first order approximation of the measurement result \( \mathbf{m} \) and the specified performance \( f \) is given by

\[
\mathbf{m} = \mathbf{m}_0 + \mathbf{G} \cdot \mathbf{s} + \mathbf{H} \cdot \mathbf{\varepsilon}_t + \mathbf{\varepsilon}_m
\]

\[
f = f_0 + \mathbf{k}^T \mathbf{s}
\]

with \( \mathbf{G} = \nabla_s \mathbf{m}, \mathbf{H} = \nabla_{\mathbf{\varepsilon}_m} \mathbf{m}, \) and \( \mathbf{k} = \nabla_s f \).

The model of Eqs. (1) and (2) includes implicit test, traditional functional test, and parameter estimation:

- For functional test, the measurement procedure aims at measuring \( f \) directly, so that \( \mathbf{G} = \mathbf{k}^T \) and \( \mathbf{m}_0 = \mathbf{f}_0 \).
- In the case of parameter estimation or fault analysis, \( f = s_i \) and therefore we set \( \mathbf{k} = \mathbf{e}_i \) and \( \mathbf{f}_0 = \mathbf{s}_0 \).

By using this model, we can therefore unify the treatment of implicit test, functional test and parameter estimation (fault analysis) in a single framework.

We assume that the estimation function \( \hat{f}(\mathbf{m}) \) should minimize the expected squared deviation from the true but unknown value \( f \),

\[
\bar{\sigma}_{\text{eff}}^2 = \mathbb{E}\{(\hat{f} - f)^2\}.
\]

The optimum estimation function is then

\[
\hat{f}(\mathbf{m}) = f_0 + \mathbf{\beta}^T \cdot (\mathbf{m} - \mathbf{m}_0),
\]

where \( \mathbf{\beta} \) is a constant vector of coefficients that still has to be determined. By applying Eq. (1), (2) and (4) to Eq. (3), we get

\[
\bar{\sigma}_{\text{eff}}^2 = \mathbb{E}\{(\mathbf{\beta}^T \mathbf{G} - \mathbf{k})^T \mathbf{s} + \mathbf{\beta}^T \mathbf{H} \mathbf{\varepsilon}_t + \mathbf{\beta}^T \mathbf{\varepsilon}_m\}^2
\]

\[
= \mathbf{\beta}^T \mathbf{G} \mathbf{C}_s \mathbf{G}^T \mathbf{\beta} + \mathbf{\beta}^T \mathbf{H} \mathbf{C}_t \mathbf{H}^T \mathbf{\beta} + \mathbf{\beta}^T \mathbf{C}_m \mathbf{\beta},
\]

because \( \mathbf{s}, \mathbf{\varepsilon}_t \) and \( \mathbf{\varepsilon}_m \) were assumed to be uncorrelated. To minimize \( \bar{\sigma}_{\text{eff}}^2 \), we want \( \nabla_{\mathbf{\beta}}(\bar{\sigma}_{\text{eff}}^2) = 0 \), and therefore

\[
\mathbf{\beta} = (\mathbf{G}^T \mathbf{C}_s \mathbf{G}^T + \mathbf{H} \mathbf{C}_t \mathbf{H}^T + \mathbf{C}_m)^{-1} \cdot (\mathbf{G}^T \mathbf{C}_s \mathbf{k}).
\]

Note that the usually ill-conditioned covariance matrices need to be treated appropriately when \( \mathbf{\beta} \) is calculated according to Eq. (6). The minimum \( \bar{\sigma}_{\text{eff}}^2 \) that can be achieved by a test based on measurement \( \mathbf{m} \) is then

\[
\bar{\sigma}_{\text{eff}}^2 = \mathbf{k}^T \mathbf{C}_s \mathbf{k} - \mathbf{\beta}^T \mathbf{G} \mathbf{C}_s \mathbf{k}
\]

\[
= \text{var}(f) - \text{var}(\hat{f}).
\]

A common measure for the quality of fit of a linear model is [22]

\[
R^2 = \frac{\mathbf{\beta}^T \mathbf{G} \mathbf{C}_s \mathbf{k}}{\mathbf{k}^T \mathbf{C}_s \mathbf{k}} = \frac{\text{var}(\hat{f})}{\text{var}(f)}.
\]

A good test design has \( \bar{\sigma}_{\text{eff}}^2 \) close to zero and \( R^2 = 1 \), whereas for a bad test that contains no information about the specified performance \( \bar{\sigma}_{\text{eff}}^2 = \text{var}(f) \) and \( R^2 = 0 \).

In the proposed model, it isn’t necessary to formulate explicit relations between \( \bar{\sigma}_{\text{eff}}^2 \) and test misclassification figures like fault
Figure 2: Leapfrog filter from [23]

<table>
<thead>
<tr>
<th>perf.</th>
<th>measurement</th>
<th>estimated</th>
<th>Monte-Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\bar{\sigma}^2_{eff}$</td>
<td>$\bar{R}^2$</td>
</tr>
<tr>
<td>$f_{3dB}$</td>
<td>Fourier only</td>
<td>2.89 Hz</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>Time-domain only</td>
<td>1.82 Hz</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>both</td>
<td>1.25 Hz</td>
<td>0.998</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Fourier only</td>
<td>0.211 dB</td>
<td>0.401</td>
</tr>
<tr>
<td></td>
<td>Time-domain only</td>
<td>0.205 dB</td>
<td>0.437</td>
</tr>
<tr>
<td></td>
<td>both</td>
<td>0.201 dB</td>
<td>0.458</td>
</tr>
</tbody>
</table>

Table 1: Comparison of linear model and Monte-Carlo simulation: $\sigma_{eff}^2$ and $R^2$ for Fourier and time domain measurements.

coverage and yield loss, because the uncertainty about the true performance that is expressed by $\bar{\sigma}^2_{eff}$ is the only cause for these misclassifications. Therefore, measurements with lower $\bar{\sigma}^2_{eff}$ remain preferable if the actual test design goal is high fault coverage and low yield loss.

Based on (7), we may calculate for a given test setup how precise an estimation of the specified performance is. Note however, that the first order approximation of Eqs. (1) and (2) does not include higher order effects in $f(s)$ and $m(s, \epsilon_t)$. The “estimation error” analyzed here is caused by measurement error and stimulus imprecision.

In the following section, we will compare the results gained by this approximation with a SPICE Monte-Carlo simulation at transistor level. To calculate $\sigma^2_{eff}$ and $R^2$ from Monte-Carlo samples, a linear regression is performed according to Eq. (4) to estimate $\beta$, $\sigma^2_{eff}$ and $R^2$.

3 Results

The leapfrog filter circuit from Fig. 2 together with a simple test setup is used to demonstrate our approach. The test input signal is a square pulse depicted in Fig. 3. The signal measured at the circuit’s output node is shown in Fig. 4. The 10kΩ resistors $R1$–$R13$ vary independently with $3\sigma_R = 5\% \cdot R_{nom} = 0.5k\Omega$, and the four capacitors $C1$–$C4$ vary independently with $3\sigma_C = 5\% \cdot C_{nom}$. Performances of the lowpass filter were the 3dB cut-off frequency $f_{3dB} \approx 1350Hz$ and the zero frequency attenuation $A_0 \approx -7dB$.
To take the precision of the output voltage measurement into account, an uncertainty of $\sigma_{\varepsilon} = 2\,\text{mV}$ was introduced. The test stimulus generation is subject to three different kinds of variations:

- $\varepsilon_{t,1}$ is a delay (jitter) of the signal, $\sigma(\varepsilon_{t,1}) = 5\,\mu\text{s}$.
- $\varepsilon_{t,2}$ is a rise time variation, $\sigma(\varepsilon_{t,2}) = 3\,\mu\text{s}$.
- $\varepsilon_{t,3}$ is a variation in the maximum input voltage, $\sigma(\varepsilon_{t,3}) = 10\,\text{mV}$.

The output signal was used to derive 20 different test measurement values $\mathbf{m}$. At first, the signal was sampled at 10 equidistant points to get 10 voltage values $v_1, \ldots, v_{10}$. Then, a Fourier transform was applied to the signal, and the magnitude of the first 10 coefficients $(c_1, \ldots, c_{10})$ were used for measurement.

### 3.1 Performance Prediction

Table 1 shows the results of a sensitivity-based calculation of $\sigma_{\text{eff}}$ and $R^2$ in comparison to a Monte-Carlo simulation on a sample of 500 circuits. The sensitivity-based results show in agreement with the Monte-Carlo validation that the estimation of $f_{3\text{dB}}$ is only slightly disturbed by a small error, whereas $A_0$ cannot be estimated very precisely. Using Fourier transform on the signal improves the estimation quality only slightly in comparison to pure time-domain measurements.

Figures 5 and 6 show $f_{3\text{dB}}$ and $A_0$ compared to their respective estimator.

### 3.2 Analysis of Influence

To identify the influence for each of the three stimulus variations (rise time, delay and maximum voltage) on the estimation of $A_0$, three additional analyses were performed. At each step, one of the three sources of variation was disabled ($\sigma_t = 0$), and then $\sigma_{\text{eff}}$ and $R^2$ were calculated. Table 2 shows, that the calculated test quality is about the same whether we consider peak voltage variation or not. In contrast to that, a reduction of the variance of the rise time and the delay would improve the estimation quality of $A_0$ very much.

### Table 2: Analysis of the influence of three types of input stimulus variation on the estimation of $A_0$. ($\sigma_t$, $\sigma_{\text{eff}}$, $R^2$)

<table>
<thead>
<tr>
<th>Variation</th>
<th>$\sigma_{\text{eff}}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise time</td>
<td>0.005 dB</td>
<td>1.000</td>
</tr>
<tr>
<td>Delay</td>
<td>0.009 dB</td>
<td>0.999</td>
</tr>
<tr>
<td>Peak voltage</td>
<td>0.130 dB</td>
<td>0.775</td>
</tr>
<tr>
<td>$\checkmark$</td>
<td>0.201 dB</td>
<td>0.458</td>
</tr>
</tbody>
</table>

### Table 3: The influence of input stimulus variation and measurement error on $A_0$ compared.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\sigma_{\text{eff}}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>157 $\Omega$</td>
<td>0.118</td>
</tr>
<tr>
<td>R2</td>
<td>157 $\Omega$</td>
<td>0.153</td>
</tr>
<tr>
<td>R3</td>
<td>157 $\Omega$</td>
<td>0.118</td>
</tr>
<tr>
<td>R4</td>
<td>159 $\Omega$</td>
<td>0.097</td>
</tr>
<tr>
<td>R5</td>
<td>144 $\Omega$</td>
<td>0.256</td>
</tr>
<tr>
<td>R6</td>
<td>150 $\Omega$</td>
<td>0.195</td>
</tr>
<tr>
<td>R7</td>
<td>145 $\Omega$</td>
<td>0.249</td>
</tr>
<tr>
<td>R8</td>
<td>153 $\Omega$</td>
<td>0.160</td>
</tr>
<tr>
<td>R9</td>
<td>151 $\Omega$</td>
<td>0.179</td>
</tr>
</tbody>
</table>

### Table 4: Parameter estimation.

To take the precision of the output voltage measurement into account, an uncertainty of $\sigma_{\varepsilon} = 2\,\text{mV}$ was introduced. The test stimulus generation is subject to three different kinds of variations:

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For the designer of BIST circuitry, this analysis shows that constant rise time and delay are more important than keeping the exact value of the peak voltage.

For Table 3, the same type of analysis was performed to compare the total influence of stimulus variation to the influence of measurement error. When both causes of error are neglected (first row), testing of $A_0$ seems to be perfectly possible. If only measurement error is taken into account, no significant degradation is observed (second row). The assumption of a perfect test stimulus therefore results in an illusively optimistic estimation of test quality. On the other hand, the assumption of an exact measurement disturbed only by test stimulus variation (third row) will give a wrong impression of the actual test quality, too.

Table 3 shows, how important it is to consider measurement error and test stimulus variation together in the same analysis.

### 3.3 Parameter Estimation and Fault Analysis

The estimated variable $f$ is not necessarily a circuit performance, but the estimation equations above may also be used for parameter estimation. Table 4 shows the estimation quality of every circuit parameter, which is remarkably bad. Figure 7 shows a plot of the capacitance of $C_4$ (the parameter with the best $R^2$ in Table 4), and its estimator.

Measurement error and input stimulus precision determine the quality of parameter estimation, too. If we assume a perfect stimulus and no measurement error, $R^2$ for $R5$ rises to 0.98 for example.

Table 4 shows that parameter estimation based on the example test setup is not possible with the given measurement error and test stimulus variation. Nevertheless Figure 5 shows that prediction of the specified performance $f_{SB1}$ is very well possible with this simple test setup. This confirms that fault analysis takes much more effort than go/no-go testing, but it is also important for choosing the test design strategy to apply to this circuit. Direct estimation of the specified performance may result in a much simpler and faster test than a test based on parameter estimation and specifications in parameter space, like $R_x min \leq R_x \leq R_x max$.

### 4 Conclusion

We proposed a new methodology to evaluate the influence of test stimulus variation on analog test. A first-order approximation of measurements and specified circuit performances was used to evaluate the influence of both test stimulus variation and measurement error on the accuracy of performance estimation by measurement. The proposed method can be applied to functional test, indirect test and to parameter estimation or fault analysis. The approximative results showed to conform to Monte-Carlo simulation at transistor level.

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### References


